NASA TN D-1025

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TECHNICAL NOTE

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RECEIVING SYSTEM DESIGN FOR THE ARRAYING OF INDEPENDENTLY STEERABLE ANTENNAS FOR DEEP SPACE COMMUNICATIONS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON April 1962

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RECEIVING SYSTEM DESIGN FOR THE ARRAYING OF INDEPENDENTLY STEERABLE ANTENNAS FOR DEEP SPACE COMMUNICATIONS

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SUMMARY

This report considers the problem of arraying independently steerable antennas for use in deep space communications systems. It describes a basic design of a receiving system capable of utilizing all the signal power received on a number of antennas without requiring that the signals be phase coherent. It also discusses the various considerations involved in the selection and location of antennas for use in such an array. This report indicates that for very large aperture systems, the array approach offers both economic and technical advantages over the single-reflector approach.

INTRODUCTION

The purpose of this report is to describe a method of obtaining ground-based receiving systems with extremely large effective antenna apertures for use with deep space probes which is both economical and technically feasible. With the indicated method, it should be possible to obtain antenna gains greater than can be obtained in a single parabolic reflecting antenna using present construction methods. The proposed system will sum the demodulated signals received by a number of independently steerable antennas. Because of the relatively low frequency of the demodulated signals, they may be summed with no appreciable phasing problems. Since in most cases it would be necessary for the antenna array system to perform to the same threshold as a single antenna receiver combination of equal area, it is necessary to design detection systems which will not degrade the signal-to-noise ratio at very low input levels.

The design of a receiver for multiple radio-frequency (designated R.F.) inputs is the basis for the design of the large antenna array. The inputs to the receiver are not phase coherent because of the different transmission times to each antenna. It is assumed, however, that the rate of change of phase difference between inputs is predictable and is primarily determined by the angular velocity of the signal source relative to the antenna array. This angular velocity would primarily be the sideral rate for deep space probes. Based on this assumption it is possible to design a receiver for multiple antennas which will be capable of performing to the same threshold as a phase-lock receiver operating from a single antenna of equal aperture.

SYMBOLS

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A	effective aperture area, sq ft
B	bandwidth constant, conventional phase-lock detector, radians/sec
B_1	bandwidth constant of primary loop, multiple input receiver, radians/sec
B_2	bandwidth constant of secondary loop, multiple input receiver, radians/sec
B_n	effective input noise bandwidth, conventional phase-lock detector, radians/sec
B_{n1}	effective input noise bandwidth of primary loop, multiple input receiver, radians/sec
B_{n2}	effective input noise bandwidth of secondary loop, multiple input receiver, radians/sec
C	capacitance, farads
c	propagation velocity, $\approx 10^9$ ft/sec
D	diameter of antenna
E_{Nn}	effective input noise voltage to nth receiver, volts
E_r	automatic gain control constant, volts
E_{Sn}	signal input voltage to nth receiver, volts
$(E_{SO}/E_{NO})^2$	ratio of signal power to noise power at receiver output
e_i	input, volts

e_o	output, volts
e_{rn}	control voltage proportional to $\left(E_{Sn}/E_{Nn}\right)^2$, volts
e_{ro}	control voltage proportional to $\left(E_N/E_S\right)^2_{av}$, volts
$F(s)$	transfer function of loop filter, conventional phase-lock detector
$F_1(s)$	transfer function of primary loop filter, multiple input receiver
$F_2(s)$	transfer function of secondary phase-lock detector, multiple input receiver
f	frequency, cps
f_d	Δf due to Doppler shift, $f_r - f_t$, cps
f_r	average received frequency, cps
f_t	transmitting frequency of signal source, cps
Δf_a	difference between frequency received at one antenna and average frequency received, cps
G	voltage gain of radio-frequency and intermediate frequency amplifiers and detector
$j = \sqrt{-1}$	
K_1	gain constant of phase detector and d-c amplifier (fig. 1), volts/radians
K_2	gain constant of d-c amplifier and secondary voltage-controlled oscillator (fig. 1), radians/sec-volt
K_3	gain constant of d-c amplifier and primary voltage-controlled oscillator (fig. 1), radians/sec-volt
n	number of channels in receiver
R_1,R_2	resistances, ohms
s	operator in Laplace transform

t	time
v	radial velocity of vehicle with respect to receiver, ft/sec
$Y(s)$	transfer function of conventional phase-lock detector, θ_{out}/θ_{in}
$Y_1(s)$	transfer function of primary loop, multiple input receiver, $\dfrac{\theta_a + \theta_{bav}}{\theta_{av}}$
$Y_2(s)$	transfer function of secondary loop, multiple input receiver, $\dfrac{\theta_{bn}}{\theta_n - \theta_a}$
$\dfrac{d\gamma}{dt}$	angular velocity of vehicle relative to receiver, radians/sec
θ_a	phase out of primary voltage-controlled oscillator, multiple input receiver, radians
θ_e	phase error, conventional phase-lock detector, $\left(\theta_{in} - \theta_{out}\right)$, radians
θ_n	phase input to nth channel, multiple input receiver, radians
θ_{av}	average phase input, multiple input receiver, radians
θ_{bn}	phase out of nth secondary voltage-controlled oscillator, multiple input receiver, radians

$$\theta_{bav} = \frac{\theta_{b1} + \theta_{b2} \cdot \cdot \cdot \theta_{bn}}{n}$$

θ_{in}	phase input, conventional phase-lock detector, radians
θ_{out}	phase output, conventional phase-lock detector, radians
λ_t	wavelength of transmitted frequency
σ	root-mean-square deviation of antenna surface from true parabolic
ω	frequency, radians/sec

ω_a free-running frequency of primary voltage-controlled oscillator, radians/sec

ω_b free-running frequency of secondary voltage-controlled oscillator, radians/sec

Dots over symbols denote derivative with respect to time.

RECEIVER DESIGN

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The difficulty in designing a receiver to operate from independently steerable antennas is due to the fact that the angular modulation appearing on each individual input is different. The modulation seen on the received signals can be grouped into two categories. The first category is that portion of the modulation which is common to all inputs. This portion includes the intentional and incidental modulation created at the transmitter plus the average Doppler shift or the Doppler shift seen at the array center. The second category is that portion of the modulation which is peculiar to each individual input. This portion includes the individual Doppler shift relative to the average Doppler shift, wave front distortion caused by atmospheric disturbances, and different phase shifts caused by amplifiers and cables. It is possible to design a system which consists of one primary phase-lock loop for the detection of the angular modulation which is common to all inputs and a number of secondary phase-lock loops for the detection of the modulation which is peculiar to each input.

Figure 1 shows the basic design of a system for detecting a number of radio-frequency inputs which are not phase coherent. Appendix A gives the derivation of the transfer functions in the receiver. The radio-frequency inputs are applied to converters where the average received phase θ_a developed in the primary voltage-controlled oscillator (designated V.C.O.) is subtracted from each signal. The resultant signals are then applied to phase detectors (secondary phase-lock loops). These detectors have a bandwidth capable of tracking only the difference between each individual received frequency and the average received frequency. The outputs of these detectors are then summed in a manner such that the resulting signal-noise ratio is equal to the sum of the input signal-noise ratios. (See appendix B). This signal is then applied to the primary voltage-controlled oscillator to complete the primary phase-lock loop. The primary loop must have a bandwidth capable of tracking the frequency modulation which is common to all inputs. This modulation includes any modulation which occurs at the transmitter plus the average Doppler shift. In the cases of amplitude modulation or narrow-band-phase modulation, this primary-loop bandwidth is determined primarily by the transmitter instability. To maintain a minimum threshold, it is necessary to maintain the signal-noise ratios in the secondary loops at a value equal to or greater than that in the primary loop. These signal-noise ratios are obtained by restricting the noise bandwidth of the secondary loops to less than that of the primary loop divided by the number of antennas employed in the system. Figure 2 shows the primary-loop noise bandwidth required for tracking normal crystal-controlled transmitter instability (ref. 1) and Doppler shifts associated with 10g acceleration. (See appendix C.) It also shows the secondary-loop noise bandwidth required for tracking the relative Doppler shift if sidereal angular velocities and a maximum difference in transmission path of 5,000 feet to any two antennas are assumed. The ratio of the primary- to secondary-loop bandwidths defines the maximum number of antennas which may be employed in the array.

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Figures 3 to 6 show complete block diagrams of a receiver utilizing two antennas. The signal from each antenna is applied to its own detector in the receiver. Each signal is delayed before it is applied to its second converter by a time approximately equal to the difference in transmission time to its antenna and to the array center. This delay is required to guarantee that the detected signals will be summed in phase; it should be emphasized that the accuracy to which it must be determined is proportional to the modulation bandwidth and not to the carrier frequency. For example, for a maximum gain degradation of 1 decibel with 100-kilocycle modulation frequency, this delay is required to an accuracy of approximately 1 microsecond. This time can be computed from the angular coordinates of the direction at which the antenna is pointed. In the second converter, the phase of the primary voltage-controlled oscillator is subtracted from each signal. The resultant signals are amplified and applied to the synchronous detectors. The outputs from these detectors are the demodulated amplitude and phase of the received signal. The amplitude-modulation (AM) signals are summed to obtain the AM output and to determine the average received signal-noise ratio. The phase-modulation (PM) signals are summed to obtain the PM output and to develop the error signal to be applied to the primary voltage-controlled oscillator.

A four-channel receiver has been simulated in an analog computer to obtain preliminary information on signal-acquisition time. With a primary-loop bandwidth B_{n1} of approximately 200 cycles per second and a bandwidth ratio B_{n1}/B_{n2} of 4, the signal-acquisition time was less than 0.1 second.

ARRAY DESIGN

There are five primary considerations in the designing of an antenna array:

(1) The angular coverage which is required

(2) The frequency range over which the system must operate

(3) The angular velocity or tracking rates which must be accommodated

(4) The total system aperture which is required and

(5) The total system cost.

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Angular Coverage

The angular coverage which is required of an antenna array will normally determine the configuration of the array. Although it is not the purpose of this paper to provide the geometrical design of arrays, figure 7 shows a possible configuration of an array located at the equator for tracking in the ecliptic. This configuration was developed to obtain the maximum number of antennas with the minimum deviation of the frequency of each received signal from the average. This condition is obtained when all the antennas are located within an ellipse with the major axis in the north-south direction. The ratio of the minor to the major axis of the ellipse should be equal to the cosine of the angle between the plane of the antennas and the ecliptic. The array shown in figure 7 is arranged to minimize the antenna blocking at azimuth angles of approximately 65° to 115° and 245° to 295°.

Frequency Range

In this paper, only parabolic reflecting antennas have been considered. There is, of course, a limit to the frequency at which such antennas are effective. This limit is determined by the magnitude of the deviations of the reflecting surface from a true parabolic surface. Figure 8 is a plot of the variation of the frequency at which maximum gain occurs with the diameter of the reflecting surface. This curve was calculated by using a ratio of root-mean-square surface deviation to diameter of 10^{-4} based on current technology. (See ref. 2.) This deviation will limit the size of the individual antennas employed in an array.

Tracking Rates

The rate at which an antenna array must be capable of tracking determines, in part, the maximum number of antennas which may be used in the array. This limitation is due to the requirement that the secondary-loop noise bandwidth be equal to or less than the primary-loop noise bandwidth divided by the number of antennas. The secondary-loop bandwidth is a function of tracking rate whereas the primary-loop bandwidth is independent of tracking rate. If the primary-loop bandwidth is large because of modulation (wide-band frequency modulation (FM)), this limit is insignificant. However, if the primary-loop bandwidth is small, such as would be encountered with narrow-band frequency modulation or amplitude modulation, this limit becomes significant. Figure 9 is a plot of the variation of the maximum number of antennas in an array with frequency, tracking rates being 0.1° per second and sidereal.

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Total Aperture

The maximum receiving aperture which may be obtained in any system is defined by the antenna size and number of antennas which may be used. Figure 10 is a curve showing the total effective aperture as a function of the individual antenna size and the number of antennas.

System Cost

If it is assumed that the cost of a parabolic reflector is proportional to the area to the 1.4 power (ref. 3) and that the electronic equipment costs (radio-frequency amplifier, receiver, etc.) are fixed, it is possible to design an array for minimum cost. If the electronic equipment costs are assumed to range from \$100,000 to \$1,000,000 at each antenna, the total cost of one antenna and receiver varies with the diameter of the antenna as shown in figure 11. The cost of an array per unit aperture area may then be determined as shown in figure 12. It can be seen from this figure that there is an optimum antenna size for purposes of economy. With the assumptions just made, the optimum antenna is 50 to 150 feet in diameter. Other considerations, however, such as operation and maintenance costs and future expansion capability may indicate that larger antennas are desirable. This cost analysis does not include fixed equipment costs, which are not a function of antenna size or number of antennas, such as post-detection data handling and recording equipment, support computing equipment, and so forth.

DISCUSSION

The major problem areas in the design and construction of this receiver appear to be first, the obtaining of multipliers and operational amplifiers with a gain bandwidth product sufficient for present-day communications systems, and second, the construction of the secondary voltage-controlled oscillators with stability compatible with the narrow bandwidth. It appears possible by using commercial amplifiers and computing elements to obtain a receiver with an information bandwidth of up to 100 kilocycles. The important factor in the stability of the secondary voltage-controlled oscillators is the drift of each oscillator with respect to the others. It appears possible to obtain relative stabilities on the order of 0.01 cycle per day.

Advantages

There are a number of advantages in the use of an array such as this over a single antenna system. These advantages are as follows:

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(1) It can be seen from figure 12 that the initial cost of a large aperture system can be reduced considerably by the arraying of small antennas. For instance, a system with an effective aperture equal to that of a 400-foot-diameter antenna may be obtained by arraying 85-foot-diameter antennas at approximately one-half the cost of a single 400-foot reflecting antenna.

(2) The gain of a parabolic reflecting antenna is limited by deviations of the reflecting surface from a perfect parabolic surface. If the assumption is made that, with normal construction practices, the surface tolerances which may be obtained are directly related to the diameter of the antenna, the maximum gain of any antenna is limited. As shown in figure 13 if it is assumed that $\sigma/D = 10^{-4}$, the gain is limited to less than 60 decibels independent of size. However, the effective gain may be increased by a factor equal to the number of antennas employed when using this array design.

(3) Since the relative phase of each received signal in the array is compensated for by the phase-lock loop in the detector, it is necessary that the incoming wave front be planar over the area of one antenna only and not over the total area. This requirement indicates that the antenna array will be capable of obtaining maximum gain on wave fronts which have been distorted in passing through the atmosphere. It is expected that the rate of change of phase due to the atmosphere will be negligible compared with the relative Doppler shift.

(4) The accuracy with which an antenna must be directed toward the signal source is directly related to the beamwidth of the antenna. Since the proposed system electronically tracks the signal within the beamwidth of the individual antennas, it does not require tracking data of the accuracy required by a single antenna of equal area.

(5) The antenna-array system includes the summation of signals on a weighted basis, where the weighting is proportional to the signal-noise ratio. This method of summation should result in a high reliability as a noisy channel will automatically reduce its own weighting and the remaining channels will still be fully operative.

(6) By the use of a dual feed (vertical and horizontal) and dual detectors at each antenna, it is possible to obtain optimum polarization at all times because the signals are combined on a weighted basis such that the signal-noise ratio out of the combiner is always equal to the sum of the signal-noise ratios into the combiner.

(7) The antenna-array system consists of a number of independent antennas, detectors, and combiners. It is possible to connect these to obtain one receiving system with maximum gain or any number of individually trackable systems with proportionally lower gains. Also, because of this modular design, it is possible to increase the effective aperture at a later date without loss of the initial investment.

Disadvantages

There are a number of disadvantages associated with this receiving system design. These disadvantages are as follows:

(1) Since the system employs phase-lock detectors, it may only be used on phase-coherent transmissions.

(2) The number and location of antennas employed will limit the angular velocity to which the system will track a signal.

(3) It does not appear to be practical to use the array for transmission of signals. Although it is theoretically possible, the problem of controlling the pattern of the array for transmission purposes would probably be more costly and less reliable than the use of a higher power transmitter and one antenna.

CONCLUSIONS

With the proposed arraying technique, it appears possible to obtain very large aperture receiving systems at a cost lower than is normally associated with large single reflecting antennas. In addition to this, such a system appears to have the following advantages over single antenna systems:

(1) Higher absolute gain is obtainable,

(2) Optimum polarization is obtained at all times,

(3) It is not necessary to have a plane wave front over the entire receiving aperture,

(4) The operating beamwidth is wider,

(5) The system should be more reliable, and

(6) The system should be more versatile.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Air Force Base, Va., December 13, 1961.

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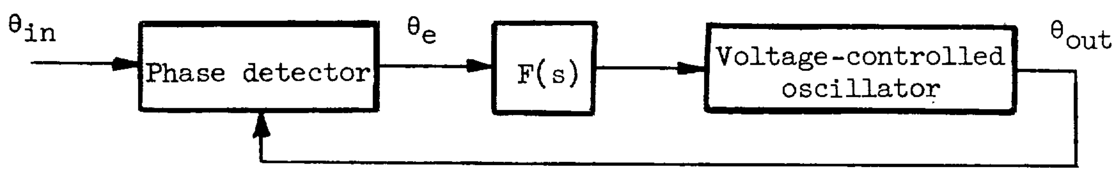
APPENDIX A

DESIGN OF A MULTIPLE-INPUT PHASE-LOCK DETECTOR

General Theory of Phase-Lock Detectors

The purpose of this section is to provide a reference in the general theory of phase-lock detector design for use in the derivation of the design equations of the phase-lock detector for multiple inputs. This section is based on work reported in references 1 and 4.

Phase-lock loop transfer function.- In the analysis of phase-lock detectors, it is convenient to describe the various signals in terms of phase angles. This may be done if it is realized that all phase angles are measured in a coordinate system which is rotating at the free-running frequency of the voltage-controlled oscillator. In the following sketch, the output of the phase detector is equal to the difference of the input phases.



$$\theta_e = \theta_{in} - \theta_{out} \tag{A1}$$

In Laplace notation, with initial conditions zero,

$$s\theta_e = s\theta_{in} - s\theta_{out} \tag{A2}$$

By definition of the voltage-controlled oscillator operation

$$\Delta\omega = s\theta_{out} = \theta_e F(s)$$

$$\theta_e = \frac{s\theta_{out}}{F(s)} \tag{A3}$$

Substituting equation (A3) into equation (A2) yields

$$\frac{s^2\theta_{out}}{F(s)} = s\theta_{in} - s\theta_{out}$$

$$\left(\frac{s}{F(s)} + 1\right)\theta_{out} = \theta_{in}$$

$$Y(s) = \frac{\theta_{out}}{\theta_{in}}(s) = \frac{\theta_{in} - \theta_e}{\theta_{in}}(s) = \frac{F(s)}{F(s) + s} \tag{A4}$$

<u>First-order phase-lock loop</u>.- It can be seen from reference 1 that the form of the desired transfer function for a first-order loop is

$$Y(s) = \frac{B}{B + S} \tag{A5}$$

where B is a constant proportional to loop bandwidth. Substituting equation (A4) into equation (A5) yields

$$\frac{F(s)}{F(s) + s} = \frac{B}{B + s}$$

$$F(s) = B \tag{A6}$$

To evaluate B, it is necessary to introduce the maximum rate of change of phase which must be tracked. It is then possible to solve for the steady-state error. (This error should not exceed 0.5 radian.)

$$\frac{\theta_{out}}{\theta_{in}} = \frac{B}{B + s}$$

$$\theta_{out} = \frac{B\theta_{in}}{B + s}$$

From equation (A1)

$$\theta_e = \theta_{in} - \theta_{out}$$

therefore

$$\theta_e = \theta_{in}\left(1 - \frac{B}{B + s}\right)$$

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With an input of ω/s^2 (phase ramp)

$$\theta_e = \frac{\omega}{s^2}\left(1 - \frac{B}{B + s}\right)$$

$$\theta_e = \frac{\omega}{s(B + s)}$$

$$\theta_e(t) = -\frac{\omega}{B}\left(e^{-Bt} - 1\right)$$

As t approaches infinity

$$\theta_e(t) = \frac{\omega}{B} \tag{A7}$$

If the steady-state error θ_e is limited to 0.5 radian

$$B = 2\omega \tag{A8}$$

The transfer function from equation (A5) is

$$Y(s) = \frac{B}{B + s}$$

or

$$\frac{\theta_{out}}{\theta_{in}}(j\omega) = \frac{1}{1 + j\frac{\omega}{B}}$$

The effective input noise bandwidth B_n by definition is:

$$B_n = \int_{-\infty}^{\infty} \left| \frac{\theta_{out}}{\theta_{in}}(j\omega) \right|^2 d\omega$$

$$B_n = \int_{-\infty}^{\infty} \frac{B^2}{B^2 + \omega^2} d\omega$$

$$B_n = \pi B \tag{A9}$$

Substituting equation (A8) into equation (A9) yields

$$B_n = 2\pi\omega \tag{A10}$$

Second-order phase-lock loop.- It can be seen from reference 1 that the form of the desired transfer function for a second-order loop is:

$$Y(s) = \frac{B^2 + \sqrt{2}Bs}{B^2 + \sqrt{2}Bs + s^2} \tag{A11}$$

where B is a constant proportional to loop bandwidth. From equation (A4)

$$\frac{F(s)}{F(s) + s} = \frac{B^2 + \sqrt{2}Bs}{B^2 + \sqrt{2}Bs + s^2}$$

Solving for $F(s)$ yields

$$F(s) = \frac{B^2 + \sqrt{2}Bs}{s} \tag{A12}$$

To evaluate B, it is necessary to introduce the maximum rate-of-change frequency to be tracked. It is then possible to solve for the steady-state error. (This error should not exceed 0.5 radian.)

$$\frac{\theta_{out}}{\theta_{in}}(s) = \frac{B^2 + \sqrt{2}Bs}{B^2 + \sqrt{2}Bs + s^2}$$

$$\theta_{out} = \theta_{in}\left(\frac{B^2 + \sqrt{2}Bs}{B^2 + \sqrt{2}Bs + s^2}\right)$$

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From equation (A1)

$$\theta_e = \theta_{in} - \theta_{out}$$

$$\theta_e = \theta_{in}\left(1 - \frac{B^2 + \sqrt{2}Bs}{B^2 + \sqrt{2}Bs + s^2}\right)$$

$$\theta_e = \theta_{in}\left(\frac{s^2}{B^2 + \sqrt{2}Bs + s^2}\right) \tag{A13}$$

With an input of $\dot{\omega}/s^3$ (frequency ramp)

$$\theta_e = \frac{\dot{\omega}}{s^3}\left(\frac{s^2}{B^2 + \sqrt{2}Bs + s^2}\right)$$

$$\theta_e = \dot{\omega}\left(\frac{1}{s\left(B^2 + \sqrt{2}Bs + s^2\right)}\right)$$

The steady-state solution is

$$\theta_e = \frac{\dot{\omega}}{B^2}$$

If θ_e is limited to 0.5 radian,

$$B^2 = 2\dot{\omega} \tag{A14}$$

The response of the loop is described by

$$\frac{\theta_{out}}{\theta_{in}}(j\omega) = \frac{1 + j\sqrt{2}\frac{\omega}{B}}{1 - \frac{\omega^2}{B^2} + j\sqrt{2}\frac{\omega}{B}}$$

The effective input noise bandwidth B_n by definition is

$$B_n = \int_{-\infty}^{\infty} \left| \frac{1 + j\sqrt{2}\frac{\omega}{B}}{1 - \frac{\omega^2}{B^2} + j\sqrt{2}\frac{\omega}{B}} \right|^2 d\omega$$

Solving for B_n

$$B_n = \frac{3\pi B}{\sqrt{2}} \tag{A15}$$

Substituting equation (A14) into equation (A15) yields

$$B_n = 3\pi\sqrt{\dot{\omega}} \tag{A16}$$

Phase-Lock Detector for Multiple Inputs

The purpose of this section of the appendix is to describe the theory of operation and obtain the design parameters of the multiple-input phase-lock detector shown in figure 1. In this analysis, the phase angles θ_a and θ_b are measured in a coordinate system which is rotating at the free-running frequencies of the oscillators ω_a and ω_b, and the phase angles θ_n describing the inputs are measured in a coordinate system which is rotating at the tuned frequency of the receiver $\omega_a + \omega_b$.

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As shown in figure 1, the input signals θ_n are applied to a converter where the output of the primary voltage-controlled oscillator, which it will be shown later is the average received phase, is subtracted from all inputs. This subtraction leaves only the difference between the individual input phase and the average received phase $\theta_n - \theta_a$ to be tracked by the secondary phase-lock loops. It can be seen from equations (A6) and (A8) that

$$K_1 K_2 = B_2 \tag{A17}$$

and

$$B_2 = 4\pi\,\Delta f_a \tag{A18}$$

The effective input noise bandwidth for purposes of determining the threshold may be obtained from equation (A10) as

$$B_{n2} = 4\pi^2\,\Delta f_a \tag{A19}$$

The transfer function of this loop may be determined as follows:

$$\frac{\theta_{bn}}{\theta_n - \theta_a} = Y_2(s) \tag{A20}$$

From equations (A5) and (A17),

$$Y_2(s) = \frac{B_2}{B_2 + s}$$

$$Y_2(s) = \frac{K_1K_2}{K_1K_2 + s} \tag{A21}$$

therefore,

$$\theta_{bn} = Y_2(s)\left(\theta_n - \theta_a\right) \tag{A22}$$

In this circuit, however, the output of the secondary loop is:

$$K_1\left(\theta_n - \theta_a - \theta_{bn}\right)$$

therefore,

$$F_2(s) = \frac{K_1(\theta_n - \theta_a - \theta_{bn})}{\theta_n - \theta_a}$$

Substituting equation (A20) into this relation yields

$$F_2(s) = K_1\left(1 - Y_2(s)\right) \tag{A23}$$

Substituting equation (A21) into equation (A23) yields

$$F_2(s) = \frac{K_1 s}{K_1K_2 + s} \tag{A24}$$

The response of the overall system may be described in terms of the average phase input minus the average phase error in the detectors divided by the average phase input:

$$Y_1(s) = \frac{\theta_{av} - \frac{1}{n}\sum\left(\theta_n - \theta_a - \theta_{bn}\right)}{\theta_{av}} \tag{A25}$$

Since

$$\frac{1}{n} \sum \theta_n = \theta_{av}$$

and

$$\frac{1}{n} \sum \theta_a = \theta_a$$

then

$$Y_1(s) = \frac{\theta_a + \frac{1}{n} \sum \theta_{bn}}{\theta_{av}} \tag{A26}$$

From equation (A22),

$$\frac{1}{n} \sum \theta_{bn} = \frac{Y_2(s)}{n} \sum \left(\theta_n - \theta_a\right)$$

$$\frac{1}{n} \sum \theta_{bn} = Y_2(s)\left(\theta_{av} - \theta_a\right)$$

Substituting this relation into equation (A26) yields

$$Y_1(s) = \frac{\theta_a\left(1 - Y_2(s)\right) + Y_2(s)\theta_{av}}{\theta_{av}}$$

It can be seen from figure 1 that

$$\theta_a = \theta_{av}\left[\frac{K_3 F_1(s) F_2(s)}{K_3 F_1(s) F_2(s) + s}\right]$$

therefore,

$$Y_1(s) = \left[\frac{K_3 F_1(s) F_2(s)}{K_3 F_1(s) F_2(s) + s}\right]\left[1 - Y_2(s)\right] + Y_2(s)$$

$$Y_1(s) = \frac{K_3 F_1(s) F_2(s) + Y_2(s) s}{K_3 F_1(s) F_2(s) + s}$$

Substituting equations (A21) and (A24) into this relation yields

$$Y_1(s) = \frac{K_1\left(K_2 + K_3 F_1(s)\right)}{K_1\left(K_2 + K_3 F_1(s)\right) + s} \tag{A27}$$

It is desirable to have this system respond in the same manner as conventional second-order phase-lock loop; therefore, this transfer function may be equated to the general transfer function of a phase-lock loop (eq. (A4)) and the filter for a second-order loop inserted for $F(s)$ (eq. (A12)), in which case,

$$\frac{B_1^2 + \sqrt{2}B_1 s}{s} = K_1\left(K_2 + K_3 F_1(s)\right)$$

and

$$F_1(s) = \frac{B_1^2 + \left(\sqrt{2}B_1 - K_1K_2\right)s}{K_1K_3 s} \tag{A28}$$

This function may be approximated by using the circuit shown in figure 5 for the loop filter. The amplifier shown is a voltage amplifier (low output impedance) with a gain of -A. The transfer function for this circuit is:

$$\frac{e_o}{e_i}(s) = \frac{1 + R_2 C s}{\frac{1}{A} + \frac{\left[R_2 + R_1(A + 1)\right]Cs}{A}}$$

If A is assumed to be very large,

$$\left[R_2 + R_1(A + 1)\right] Cs \gg 1$$

and

$$R_2 \ll R_1(A + 1)$$

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then

$$\frac{e_o}{e_i}(s) \approx \frac{1 + R_2 Cs}{R_1 Cs}$$

Equating this relation to $F_1(s)$ yields

$$R_1 C = \frac{K_1 K_3}{{B_1}^2} \tag{A29}$$

and

$$R_2 C = \frac{\sqrt{2}}{B_1} - \frac{K_1 K_2}{{B_1}^2} \tag{A30}$$

From equation (A16) the primary-loop noise bandwidth B_{n_1} is

$$B_{n_1} = 3\pi\sqrt{2\pi\dot{f}} \tag{A31}$$

From equation (A14) the bandwidth constant B_1 is

$$B_1 = \sqrt{4\pi\dot{f}} \tag{A32}$$

This system may now be designed by using the following equations:

$$K_1K_2 = 4\pi\ \Delta f$$

$$B_1 = \sqrt{4\pi\dot{f}_t}$$

$$R_1C = \frac{K_1K_3}{{B_1}^2}$$

$$R_2C = \left(\frac{\sqrt{2}}{B_1} - \frac{K_1K_2}{{B_1}^2}\right)$$

and to guarantee that the threshold does not normally occur in the secondary loops before it occurs in the primary loop, it is necessary to restrict the secondary-loop noise bandwidth to $1/n$ times the primary-loop noise bandwidth:

$$B_{n_2} \leqq \frac{B_{n_1}}{n}$$

$$\frac{B_{n_1}}{B_{n_2}} \geqq n$$

Substituting equations (A9) and (A15) into this function yields:

$$\frac{B_2}{B_1} \leqq \frac{3}{n\sqrt{2}}$$

APPENDIX B

SUMMATION FOR MAXIMUM SIGNAL-NOISE RATIO

The purpose of this appendix is to show that the indicated method of signal summation will result in an output signal-noise ratio equal to the summation of the input signal-noise ratios.

Figures 4 and 5 show the control and summation circuitry. The overall gain of the radio-frequency and intermediate frequency amplifiers is maintained at a value G such that the root-mean-square signal voltage out of the amplitude detector is constant at some value E_r:

$$E_S G = E_r$$

$$G = E_r / E_S$$

The root-mean-square noise out of the detectors therefore is:

$$E_N G = \frac{E_N}{E_S} E_r$$

A sample of this noise is taken in the noise filter and detector such that the output e_r is a d-c voltage equal to the square of the input noise voltage:

$$e_r = \left(\frac{E_N}{E_S}\right)^2 E_r{}^2$$

and e_r therefore is proportional to noise-signal ratio. The summation control voltage e_c is obtained by dividing a d-c voltage e_{ro}, which is proportional to the average N/S, by e_r.

$$e_c = \frac{e_{ro}}{e_r} = \frac{e_{ro}}{E_r{}^2}\left(\frac{E_S}{E_N}\right)^2$$

The root-mean-square signal out of the control multiplier is:

$$E_r e_c = \frac{e_{ro}}{E_r}\left(\frac{E_S}{E_N}\right)^2$$

The root-mean-square noise out of the control multiplier is:

$$E_N G e_c = \frac{e_{ro}}{E_r}\left(\frac{E_S}{E_N}\right)$$

The root-mean-square signal obtained by the summation of n-channels may be determined. Since the signals are phase coherent, it is the algebraic summation of the outputs of all multipliers or

$$\frac{e_{ro}}{nE_r}\left[\left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . + \left(\frac{E_{Sn}}{E_{Nn}}\right)^2\right]$$

The root-mean-square noise obtained by the summation of n-channels may be determined. Since the noises are not phase coherent, it is the quadratic summation of the outputs of all multipliers or

$$\frac{e_{ro}}{nE_r}\left[\left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . + \left(\frac{E_{Sn}}{E_{Nn}}\right)^2\right]^{1/2}$$

The S/N out of the signal combiner may be obtained by squaring the individual signal and noise terms and then dividing:

$$\left(\frac{E_{S0}}{E_{N0}}\right)^2 = \left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . + \left(\frac{E_{Sn}}{E_{Nn}}\right)^2$$

The control voltage e_{ro} is obtained by employing a feedback system which forces the filtered signal term to equal 1. Thus,

$$\frac{e_{ro}}{nE_r}\left[\left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . \left(\frac{E_{Sn}}{E_{Nn}}\right)^2\right] = 1$$

$$\frac{1}{n}\left[\left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . \left(\frac{E_{Sn}}{E_{Nn}}\right)^2\right] = \left(\frac{E_S}{E_N}\right)^2_{av}$$

$$e_{ro} = \frac{nE_r}{\left(\frac{E_{S1}}{E_{N1}}\right)^2 + \left(\frac{E_{S2}}{E_{N2}}\right)^2 \ . \ . \ . \left(\frac{E_{Sn}}{E_{Nn}}\right)^2} = E_r\left(\frac{E_N}{E_S}\right)^2_{av}$$

APPENDIX C

SOURCES OF FREQUENCY MODULATION OF THE RECEIVED SIGNALS

The purpose of this appendix is to indicate the amount of frequency modulation existing on a received carrier. This frequency modulation will determine the bandwidths required in the primary and secondary loops of the receiver, which determine the theoretical maximum number of antennas which may be employed in an array.

Transmitter Instability

Experiments (ref. 1) have indicated that the primary-loop bandwidth B required to track the instability of a normal crystal-controlled transmitter with a second-order phase-lock loop is $B = 4\sqrt{f_t}$ mc. From appendix A (eqs. (A31) and (A32)) this relation indicates that the effective input noise bandwidth B_{n1} required is:

$$B_{n1} = 26.7\sqrt{f_t}$$

where B_{n1} is measured in radians per second and f_t is in megacycles.

Doppler Effect

The primary-loop bandwidth required to track the rate of change of frequency due to the Doppler effect may be calculated as follows:

$$f_r = \frac{cf_t}{c \pm v}$$

$$f_d = f_r - f_t$$

$$f_d = \frac{f_t(\mp v)}{c \pm v} \approx \frac{v}{c}f_t$$

$$\dot{f}_d \approx \frac{f_t}{c}\frac{dv}{dt}$$

where

f_t	transmitted frequency
f_r	average received frequency
f_d	Δf due to Doppler
c	velocity of transmission
v	velocity of vehicle radial to receiver

If $\frac{dv}{dt} = 10g$,

$$\dot{f}_d = 0.32 f_t \text{ megacycles}$$

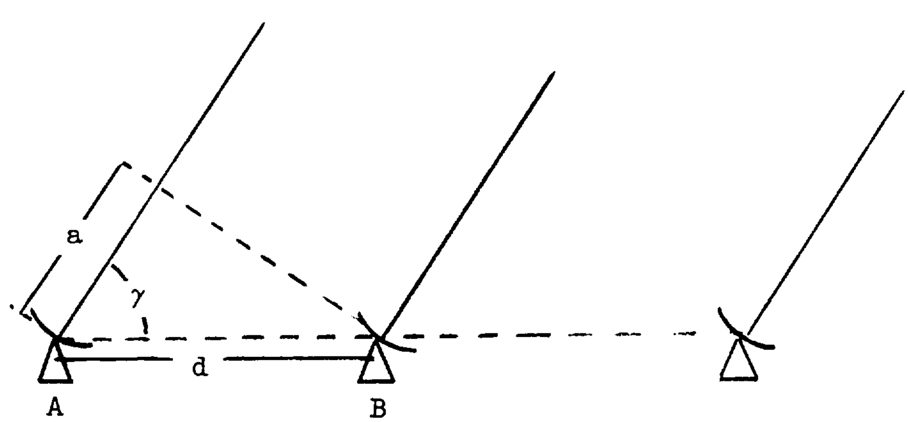
From equation (A32)

$$B_{nl} = 13.3\sqrt{f_t}$$

where B_{nl} is in radians per second and f_t is in megacycles.

Angular Velocity

The secondary-loop bandwidth required to track the relative Doppler shifts in an array may be calculated as follows:



In the preceding sketch, the difference a in path length over which the signal travels to antenna A and the antenna field center (point B) is

$$a = d \cos \gamma$$

The angular difference in received signals ϕ in cycles is:

$$\phi = \frac{a}{\lambda_t}$$

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$$\lambda_t = \frac{c}{f_t}$$

$$\phi = \frac{df_t}{c} \cos \gamma$$

The difference frequency $\frac{d\phi}{dt}$ is

$$\frac{d\phi}{dt} = \Delta f_a = \frac{df_t}{c} \sin \gamma \frac{d\gamma}{dt}$$

$$(\Delta f_a)_{max} = \frac{df_t}{c} \frac{d\gamma}{dt}$$

If d is assumed to be 2,500 feet and $d\gamma/dt$ equals 0.73×10^{-4} radians per second (sidereal)

$$\Delta f_a = \left(1.8 \times 10^{-4}\right) f_t$$

where Δf_a is in cycles per second and f_t is in megacycles. From equation (A19)

$$B_{n2} = \left(7.1 \times 10^{-3}\right) f_t$$

where B_{n2} is in radians per second and f_t is in megacycles.

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4. Gruen, Wolf J.: Theory of AFC Synchronization Proc. Inst. Radio Engineers, vol. 41, no. 8, Aug. 1953, pp. 1043-1048.

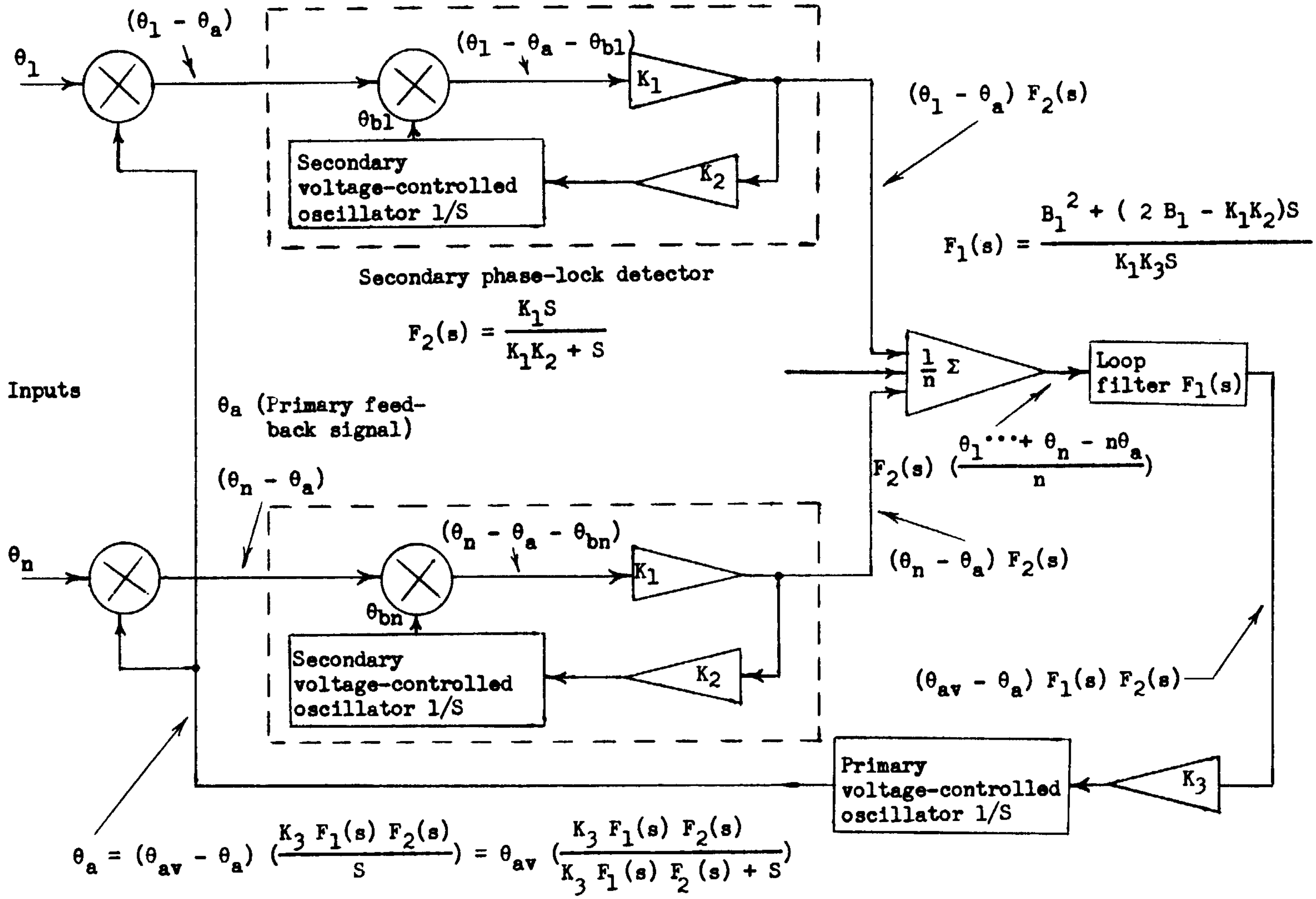

Figure 1.- Primary and secondary phase-lock loops, multiple input receiver.

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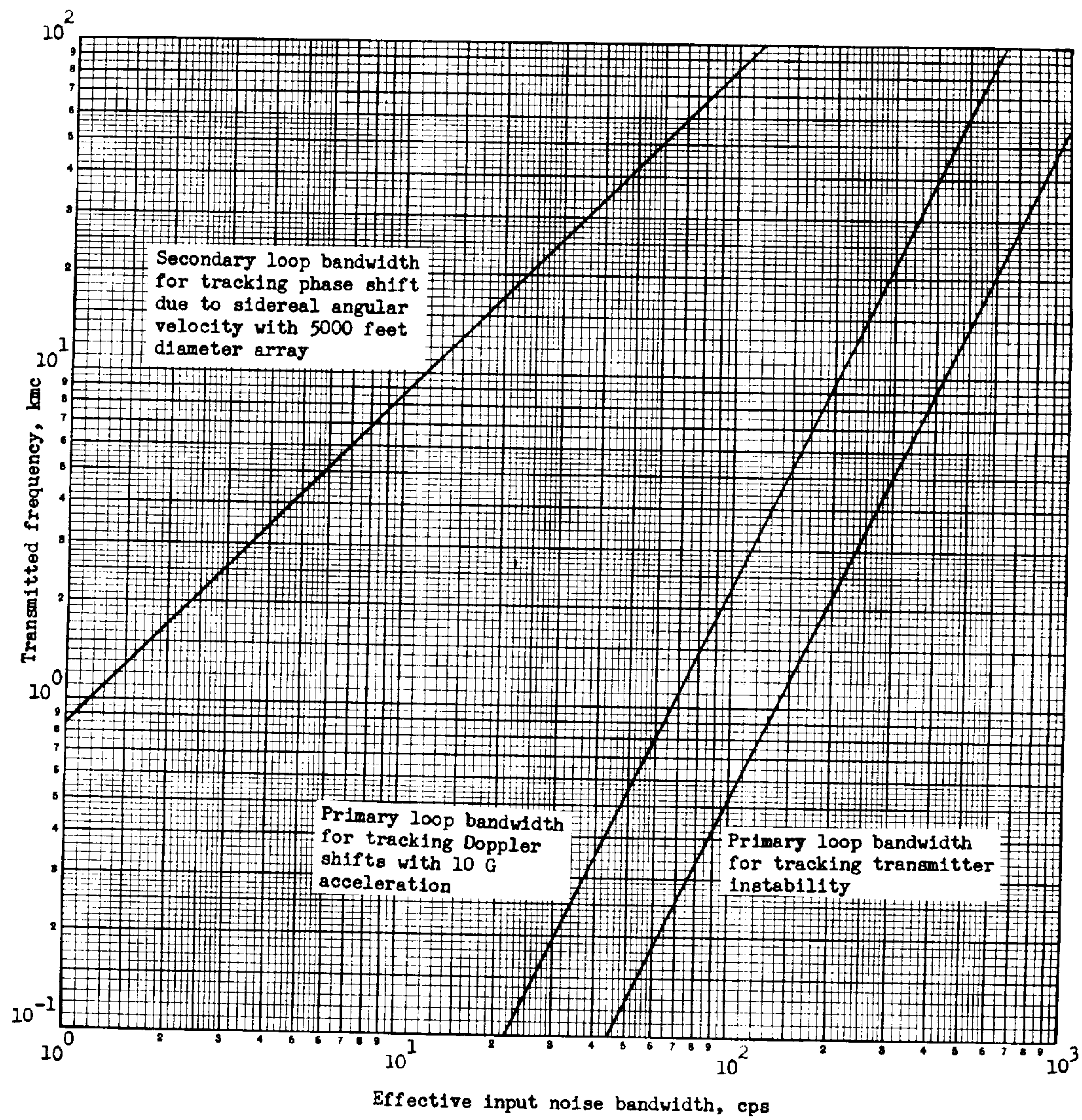

Figure 2.- Effective noise bandwidth required to track various sources of frequency shift.

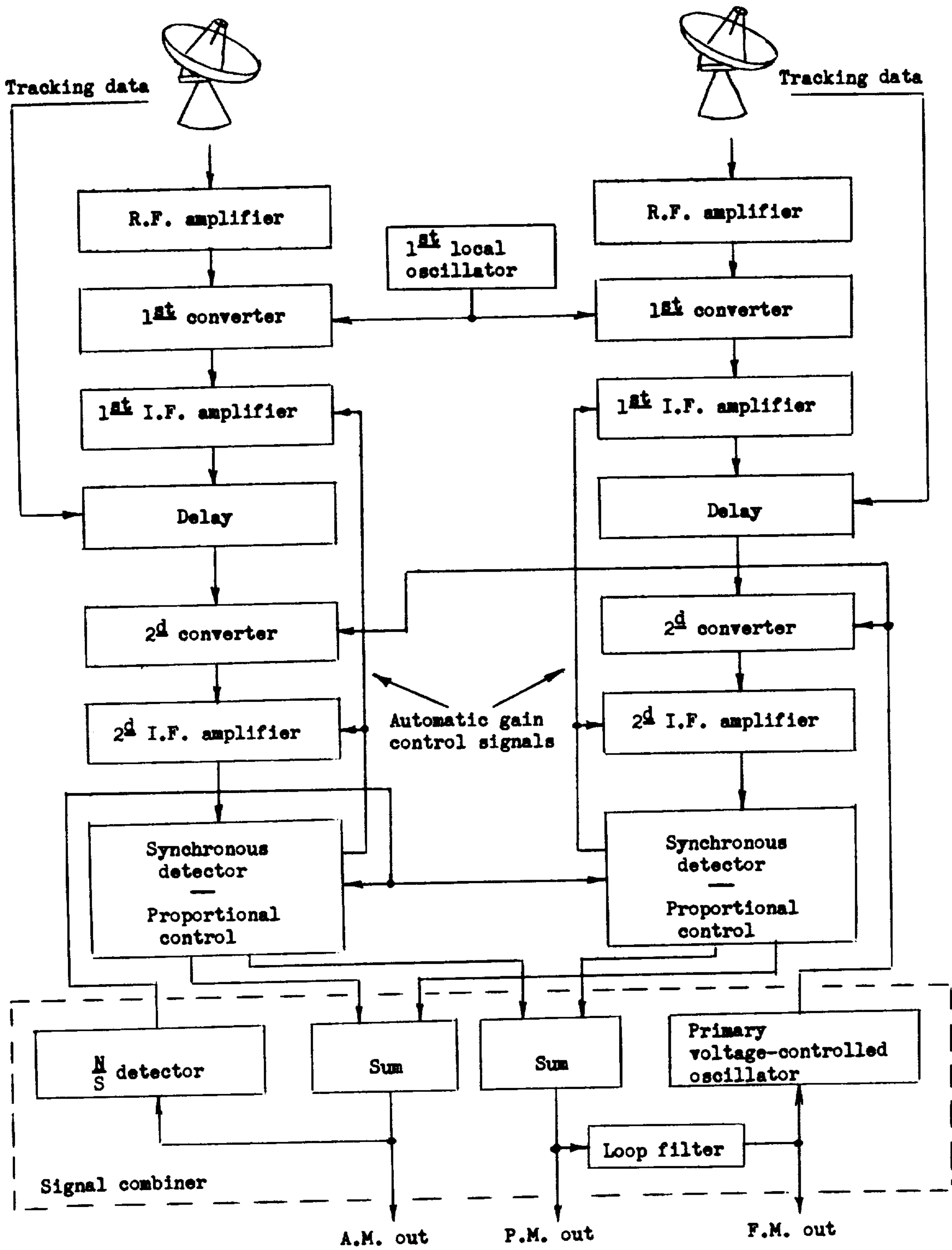

Figure 3.- Block diagram for two-channel receiver.

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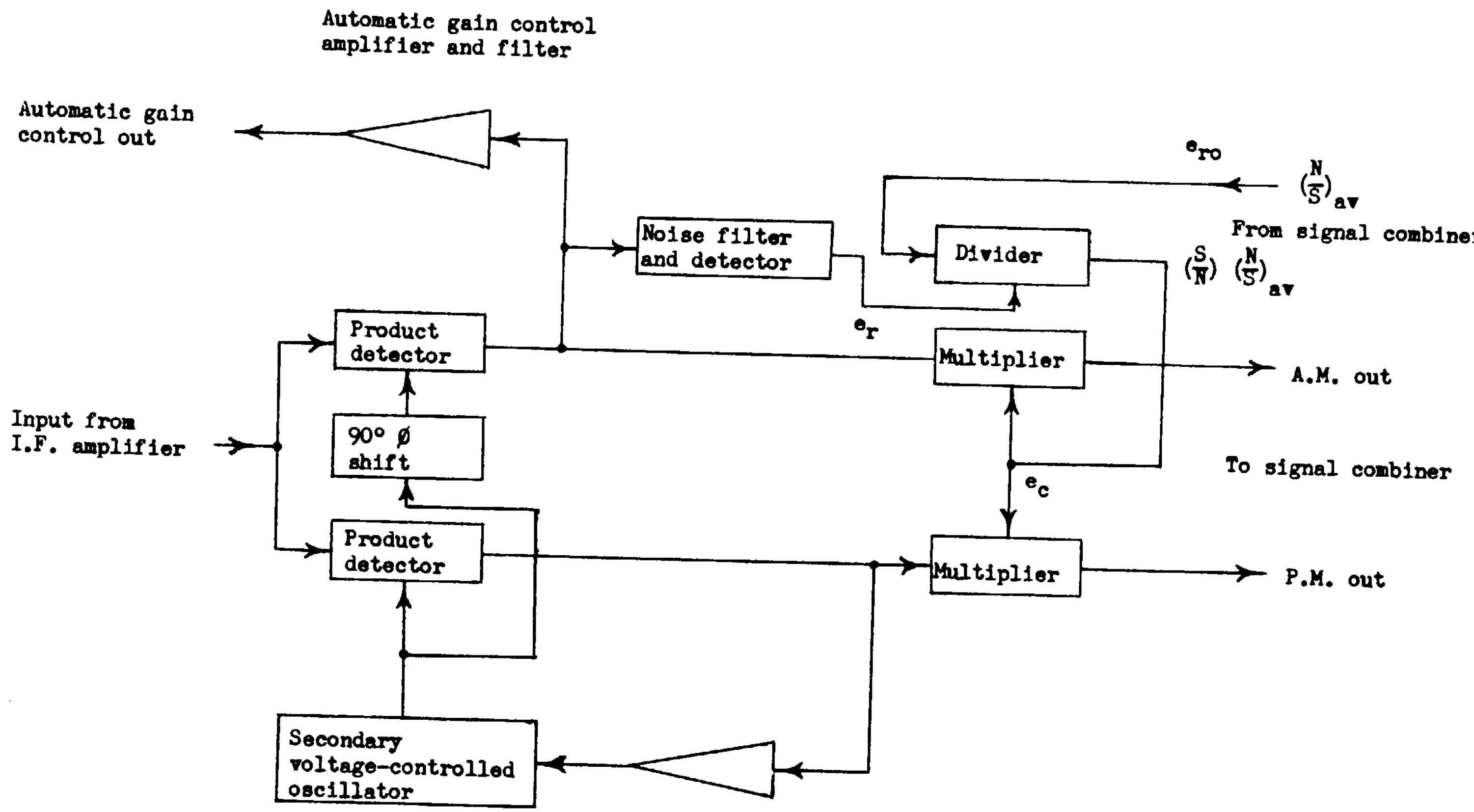

Figure 4.- Synchronous detector and proportional control.

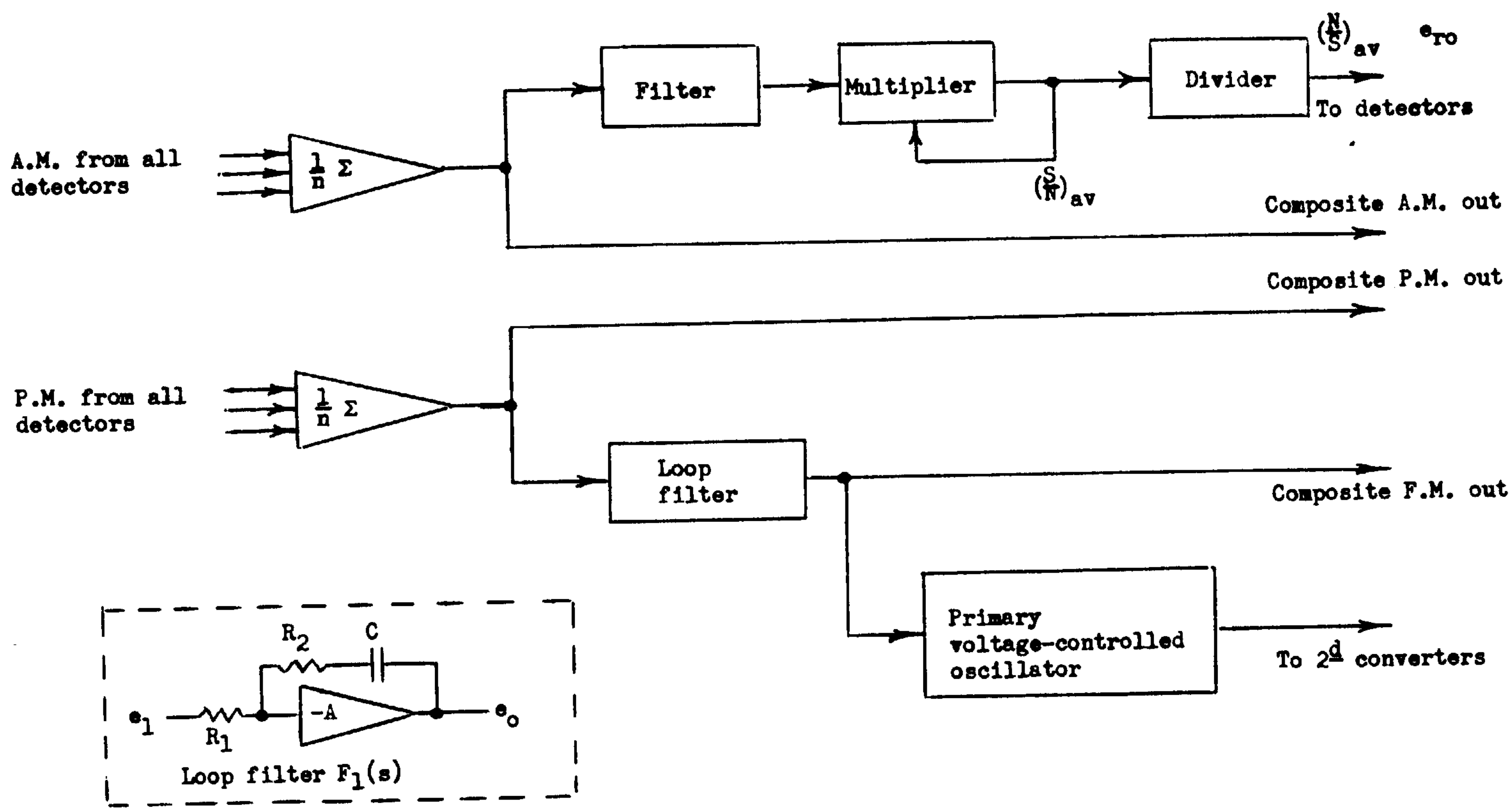

Figure 5.- Signal combiner.

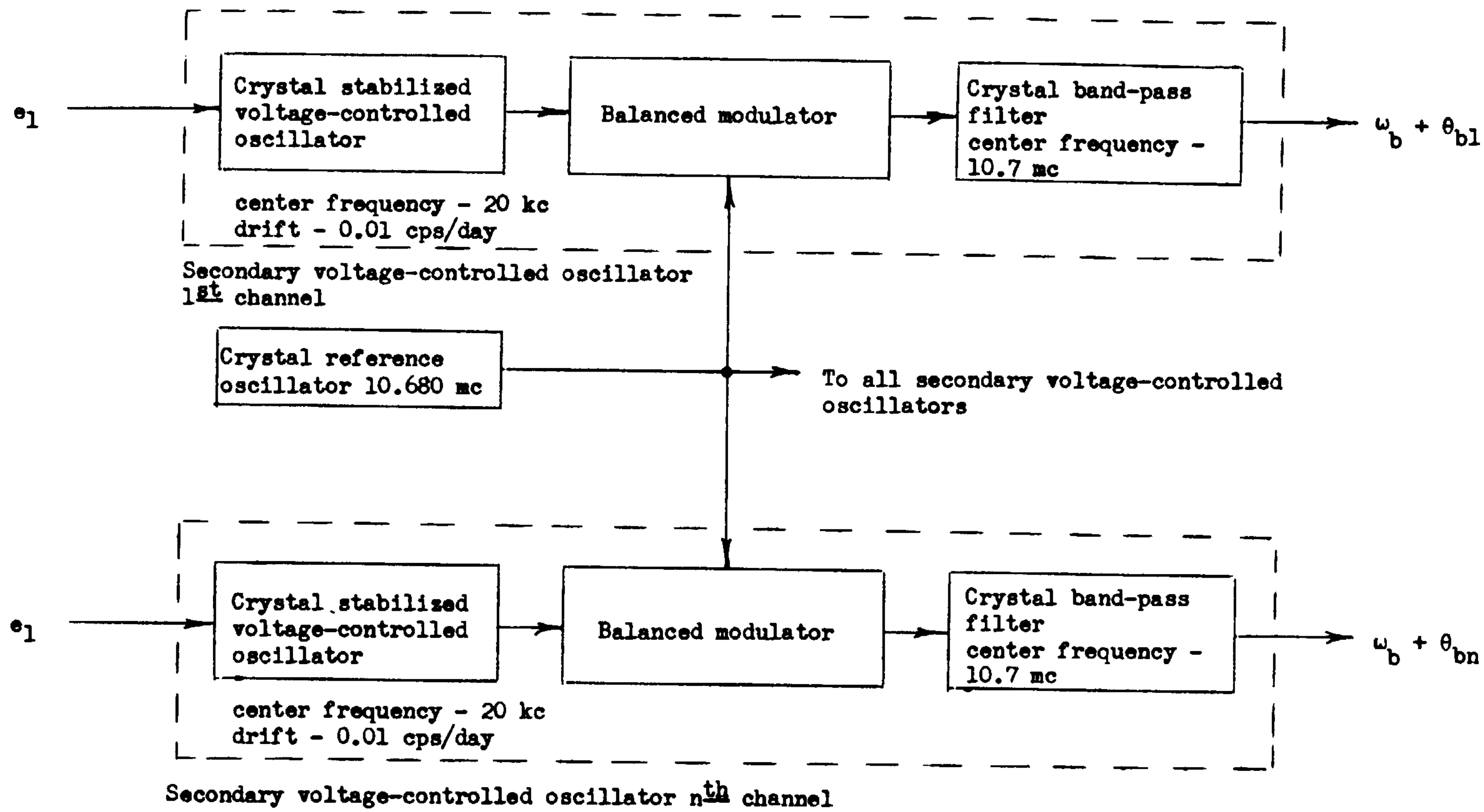

Figure 6.- Secondary voltage-controlled oscillator.

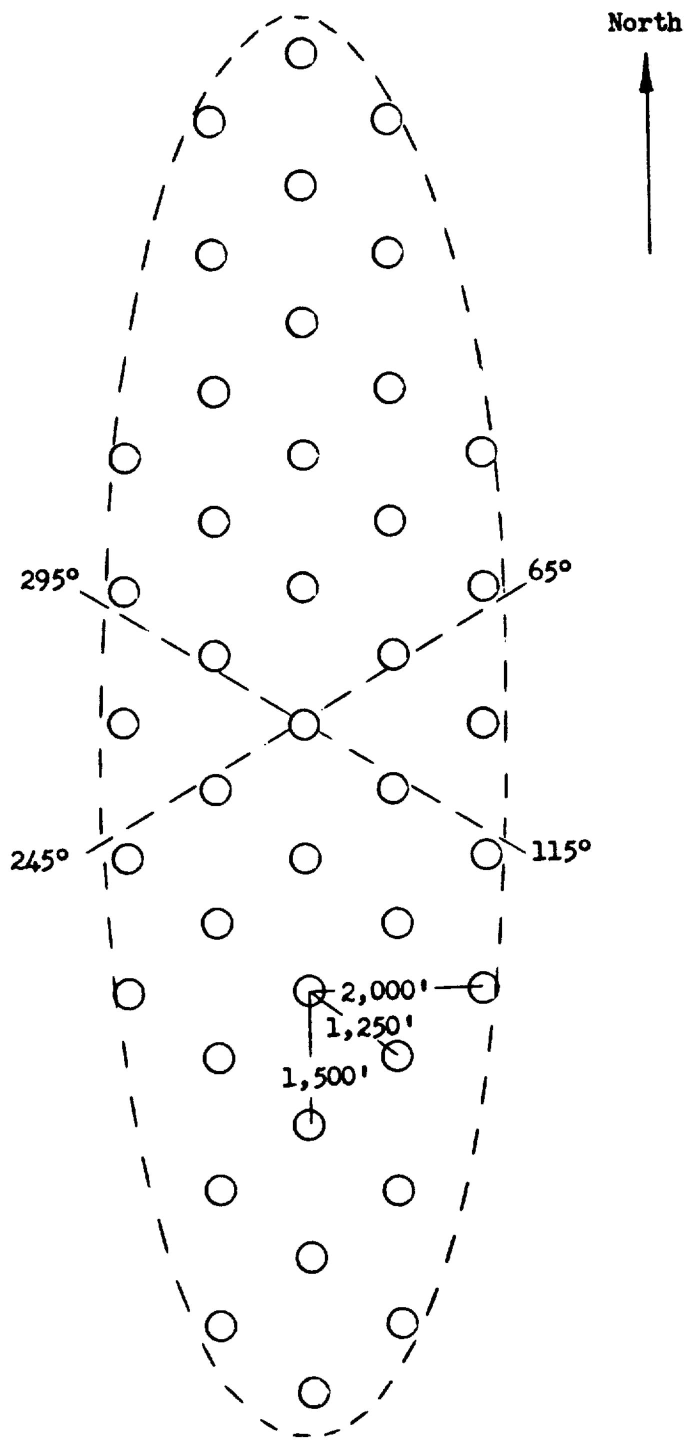

Figure 7.- Antenna array configuration for tracking in the ecliptic plane (equatorial location).

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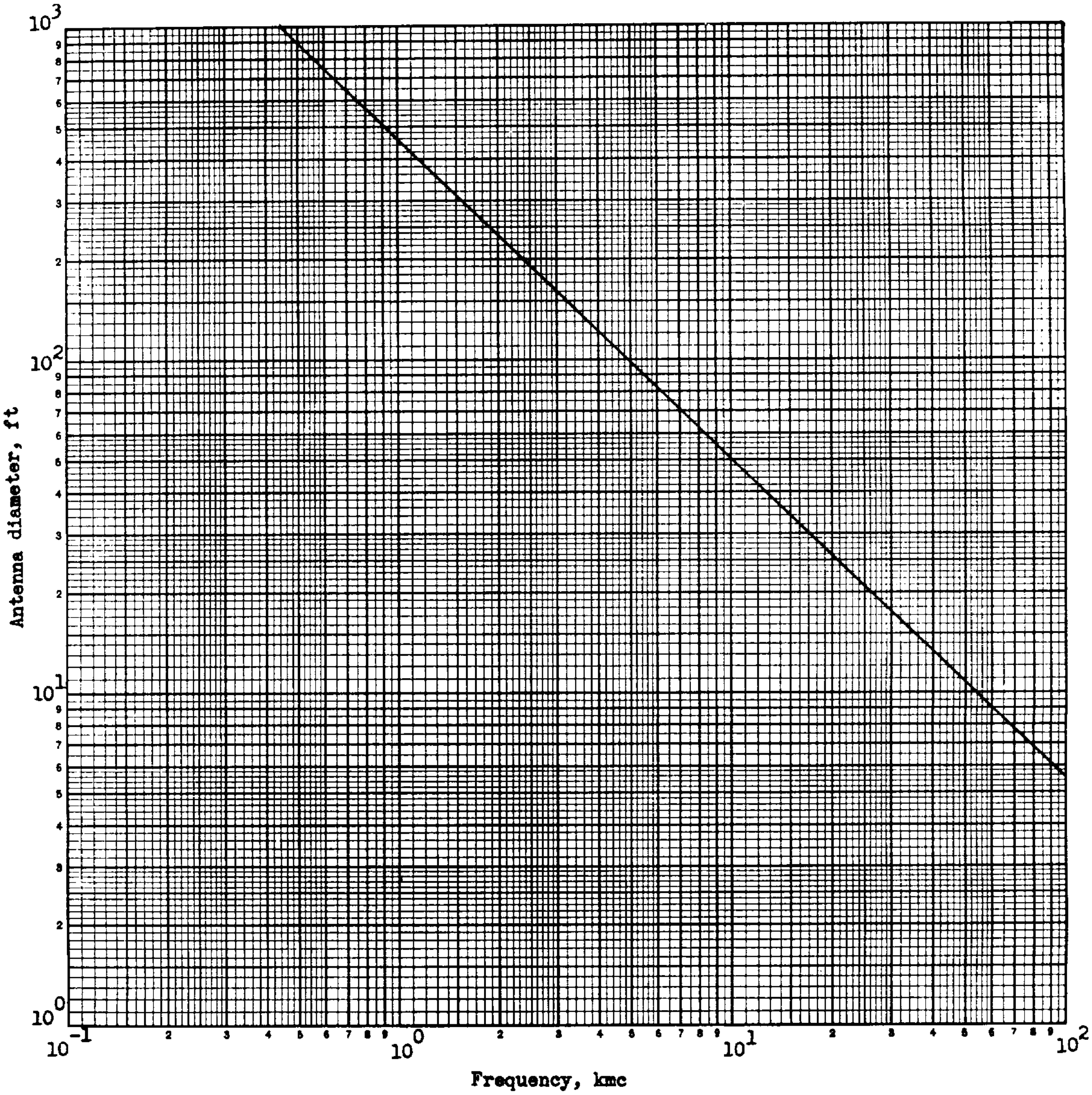

Figure 8.- Variation of frequency at which maximum gain occurs with antenna diameter. $\sigma/D = 10^{-4}$.

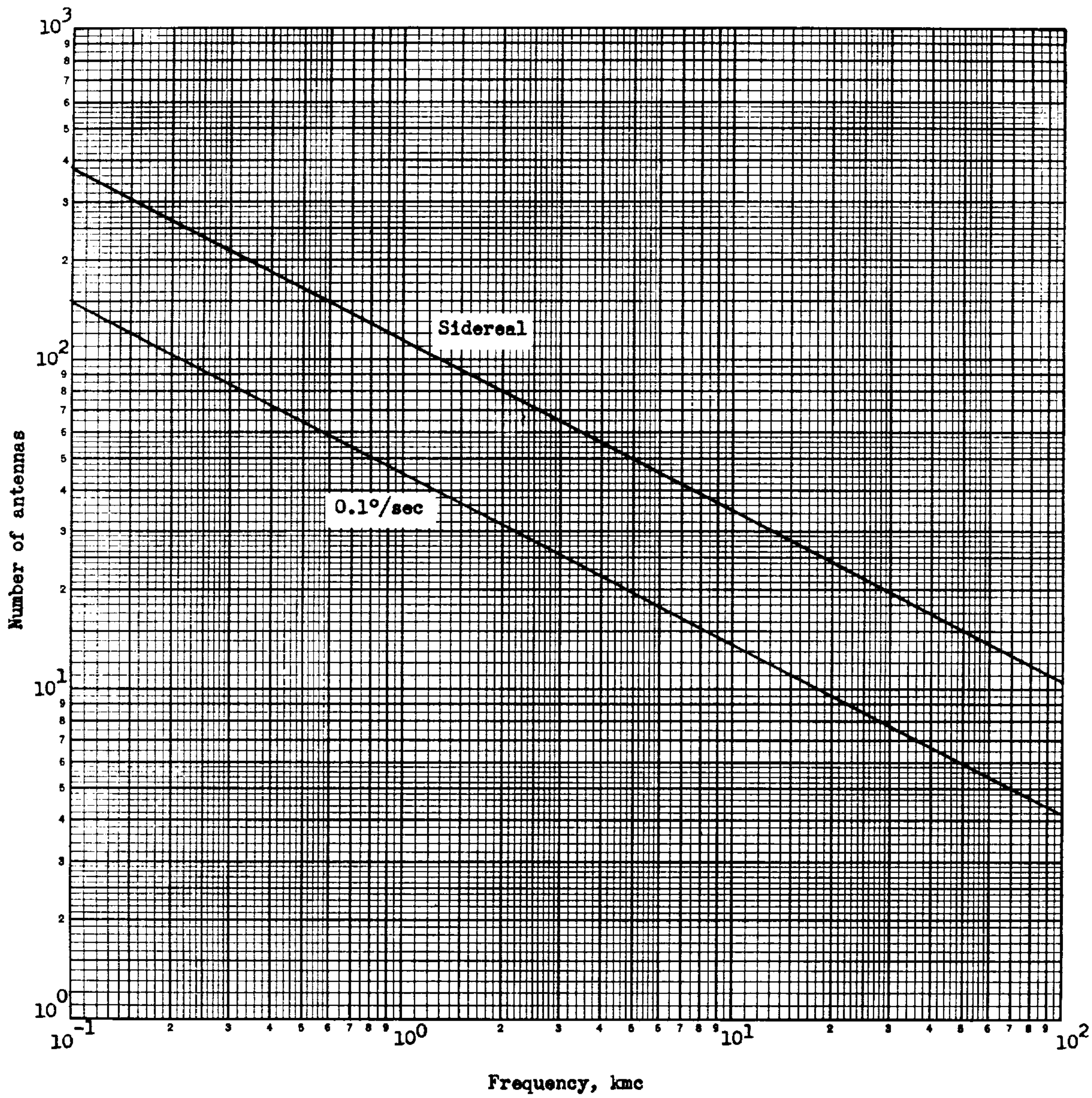

Figure 9.- Variation of maximum number of antennas in an array with frequency and tracking rate (based on maximum difference in transmission path of 5,000 feet to any two antennas).

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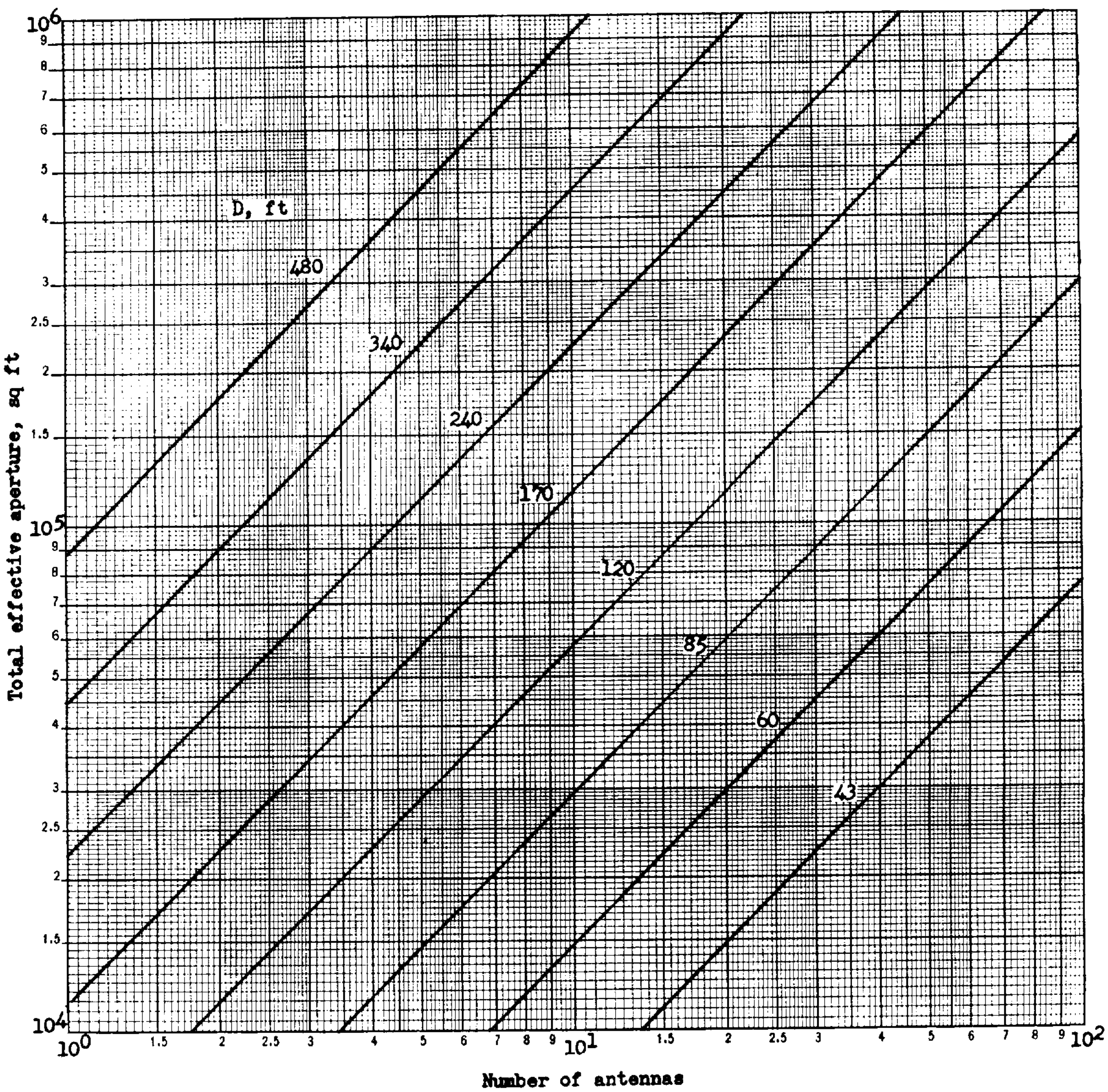

Figure 10.- Variation of effective aperture with number and diameter of antennas in array. $A \approx \frac{n\pi D^2}{8}$.

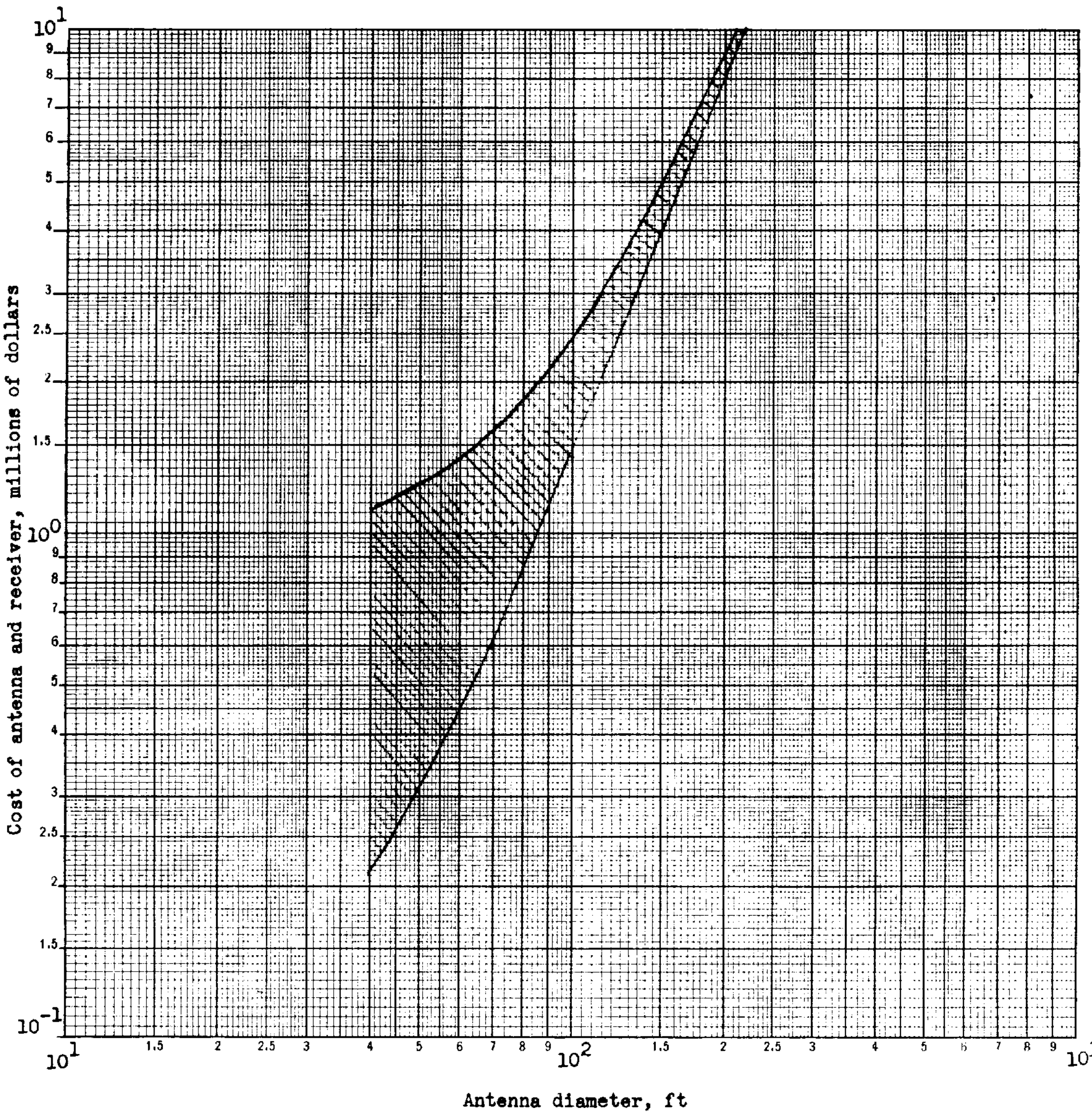

Figure 11.- Variation of estimated cost of a single steerable antenna and receiver with antenna size.

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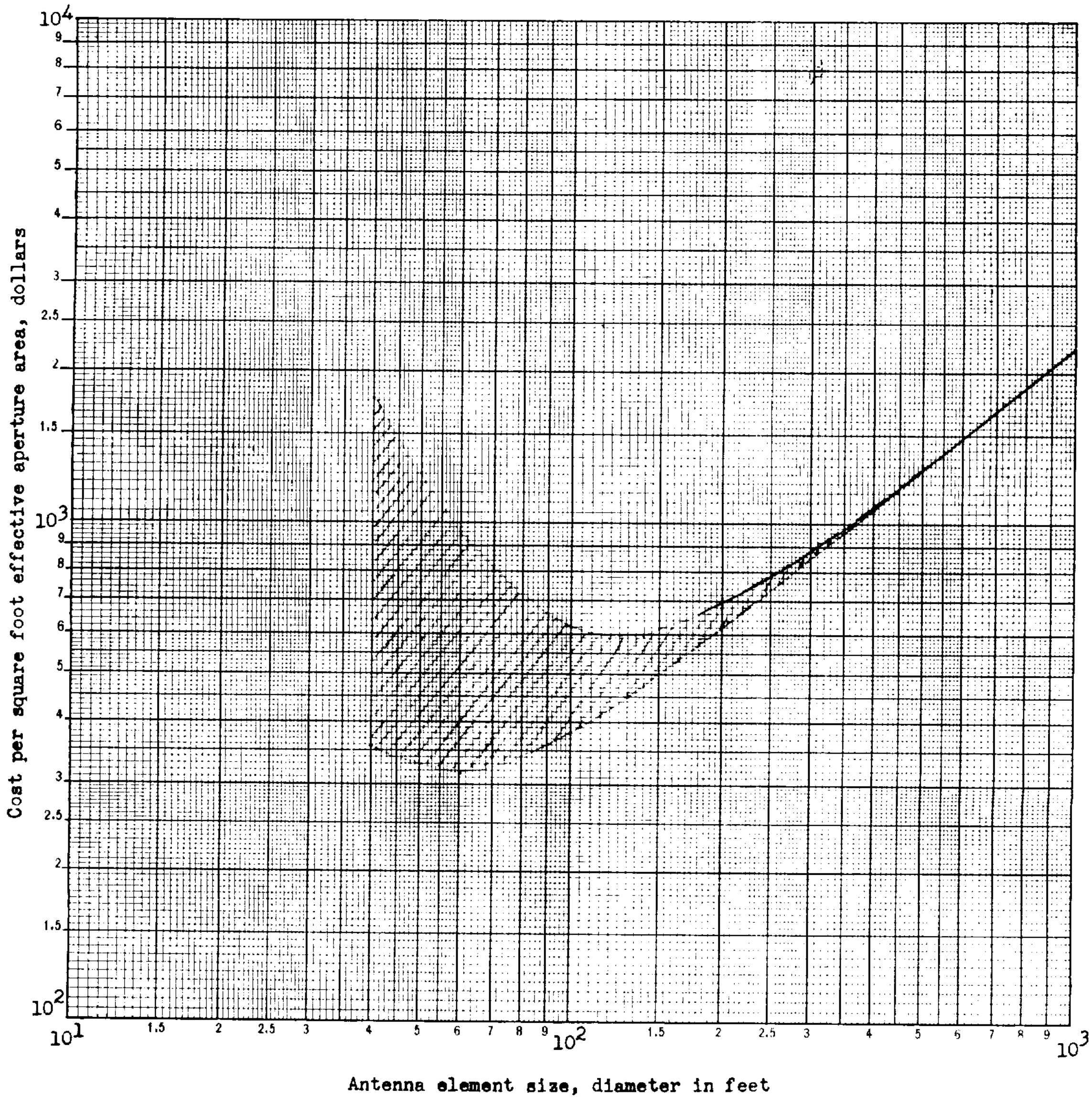

Figure 12.- Comparison of cost per unit aperture area with antenna size (based on single antenna receiver cost as shown in fig. 11).

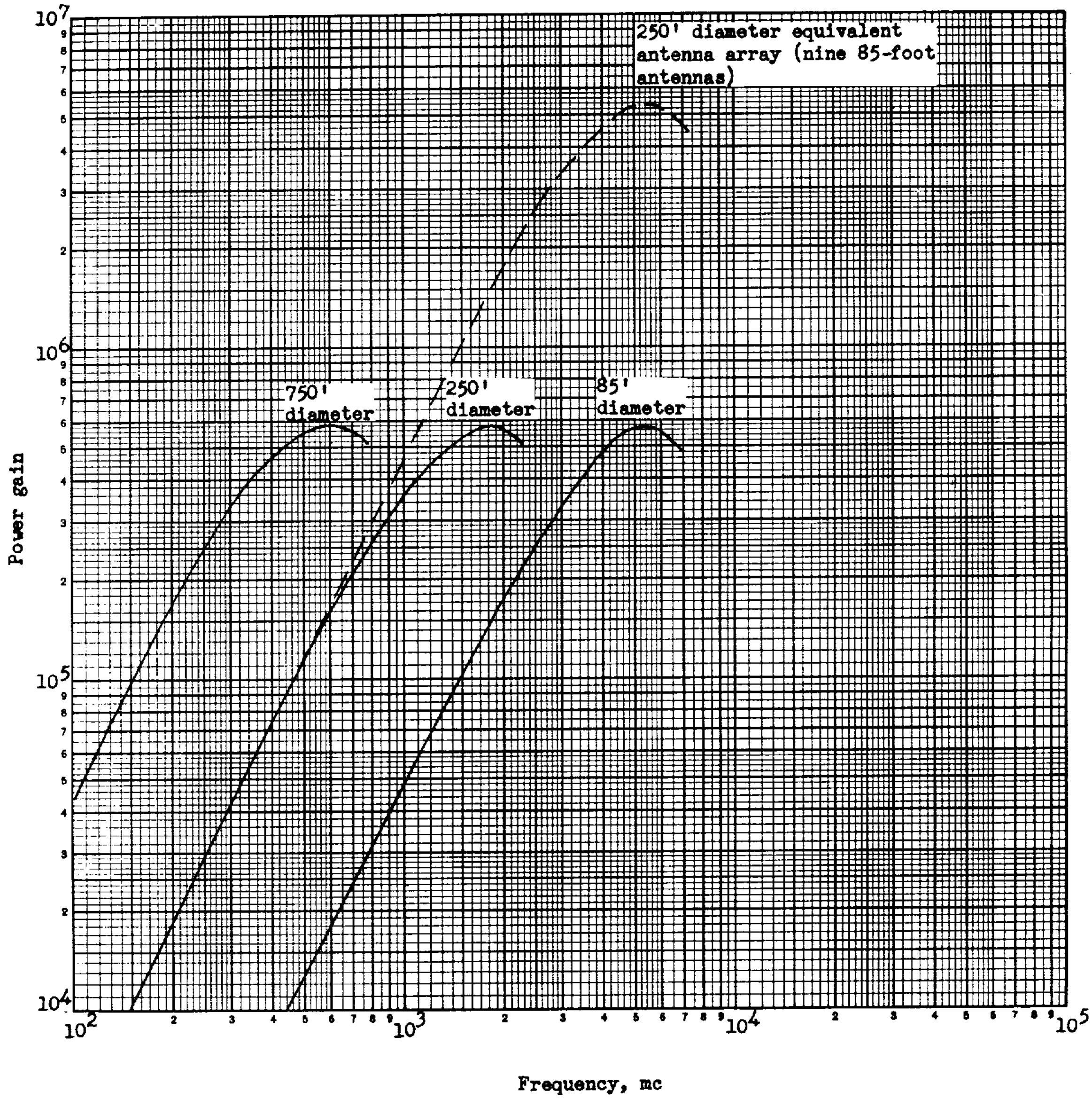

Figure 13.- Gain characteristic array and single antennas.

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NASA-Langley, 1962 L-1746